INTERACTION OF A JET-SLOT OSCILLATOR WITH A DEEP CAVITY RESONATOR AND ITS CONTROL

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The coupling mechanism between a jet-slot oscillator and a deep cavity resonator as well as the effect of feedback control on this coupling mechanism are investigated experimentally. Strong nonresonant and resonant oscillations have been observed. The nonresonant oscillations are generated by the natural instability of the jet-slot oscillator, in which the jet oscillates symmetrically. However, when resonant oscillations of the cavity are initiated, the jet switches to an antisymmetric mode of oscillation. This mode switching may be accompanied by an abrupt jump in the oscillation frequency. The paper focuses on the nature of these different types of flow oscillations and on the response of this complex flow oscillator as the resonant oscillations are gradually suppressed by means of a *simple* feedback controller. It is shown that the resonant oscillations can be attenuated by a maximum amount ranging from 11 to 13 dB. Further attenuation results in destabilization of the natural instability mode of the jet-slot oscillator. A method for improving the performance of the control system is suggested.

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1. INTRODUCTION

FLOW-EXCITED ACOUSTIC RESONANCES are often encountered in many engineering applications involving internal or external flows. The acoustic pressure associated with such resonances can cause severe noise problems and, in some cases, endanger the structural integrity of the installation. Flow-excited acoustic resonance can occur in piping systems conveying gases or liquids, flow control devices, turbomachines, boilers, heat exchangers, cavities in fast moving vehicles, and many other applications. In many of these examples, the resonance is excited by an unstable separated flow, such as a shear layer, a jet, or a bluff-body wake. Rockwell & Naudascher (1979) classified this excitation mechanism as fluid-resonant in contrast to the fluid-dynamic mechanism, which generates self-sustained oscillations of impinging shear flows in the absence of resonance effects. In the former, the fluid-resonant mechanism, the feedback event, which sustains the oscillation, is provided by the resonant sound field which excites the unstable flow at its separation location. In the latter case, the fluid-dynamic mechanism, the distortion of the vorticity field upon flow impingement produces new disturbances at the upstream separation location. Although the fluid-dynamic mechanism by itself can generate strong flow oscillations, its coupling with an acoustic mode drastically increases the oscillation amplitude. This paper considers the coupling mechanism between a jet-slot flow, which is a robust flow oscillator, and a deep cavity resonator. Special attention is given to the changes which may occur in the instability of an impinging jet when an acoustic resonance develops or subsides. As will be shown in this

study, the occurrence of resonance is associated with a fundamental change in the mode of jet instability, and in some cases, also with a jump in the frequency of oscillation.

Active control of flow-excited acoustic resonance has received considerable attention in recent years. Ffowcs Williams & Huang (1989), Huang & Weaver (1991) and Welsh *et al.* (1991) have used loudspeakers to counteract the resonant sound field of different resonators and thereby suppress the resonant oscillations. More recently, active suppression of cavity acoustic resonance has been demonstrated, with a varying degree of success, by means of perturbing the shear layer at its separation location with the aid of oscillating flaps (McGrath & Shaw 1996), pulsed mass injection (Sarno & Franke 1994) or piezoelectric actuators (Cattafesta *et al.* 1997). In these studies, the excited acoustic modes consisted of standing waves along the cavity length, i.e. in the direction of the flow. It is noteworthy that since these resonances were excited by a turbulent grazing flow, suppression of the resonant oscillations did not destabilize other instability modes of the impinging shear layer.

In addition to the coupling mechanism mentioned above, the present study also considers the effect of feedback control on the resonant oscillations of two well-tuned, deep cavities in the presence of an additional flow oscillator, which is capable of generating strong non resonant oscillations. The nature of this excitation mechanism is clearly more complex than those considered in the above-mentioned studies and is therefore more difficult to control. The loudspeakers used for control in this study are focused on the jet exit only and therefore have little effect on the resonant acoustic field. This approach has been successfully employed by Ziada (1995) to suppress the natural oscillations of the jet-slot and the jet-edge oscillators in the absence of resonance effects. However, in the present study, a rather simple controller is used in order to understand the system behaviour, as the resonant oscillation, which is shown to be very different in nature from the natural oscillation of jet-slot case, is gradually attenuated. This understanding is essential to be able to design a controller that is capable of stabilizing all plausible instability modes of this complex flow oscillator. Three cases are considered: (a) jet-slot oscillations without acoustic resonance effects; (b) cavity resonance when the frequency of the natural jet-slot oscillation is nearly equal to the cavity resonance frequency; (c) cavity resonance when the frequency of the natural jet-slot oscillation is substantially different from the resonance frequency. In the latter two cases, the pattern of jet oscillation at resonance is fundamentally different from that of the natural jet-slot oscillation. These cases are therefore distinct from those considered in previous studies.

2. EXPERIMENTAL SET-UP

The tests were carried out in a large anechoic chamber. As shown in Figure 1, the test facility comprised a jet-slot system (i.e., a planar jet impinging on a slot in a flat plate) which is combined with an acoustic resonator consisting of two well-tuned, deep cavities. The jet was produced by means of a two-dimensional nozzle and the slot was formed by two plates. Both the nozzle exit and the slot had a width of 25 mm and a length of 420 mm, i.e., the dimension normal to the plane of Figure 1 was 420 mm. The plates forming the slot, the front plates of the nozzle and four sidewalls (items 4, 5 and 6 in Figure 1) formed two identical deep cavities, symmetrically positioned with respect to the nozzle and the slot. Since the two cavities are well tuned, they can generate strong resonances of the standing acoustic waves along their depths (Ziada & Bühlmann 1992; Ziada 1993; Peters 1993). The nozzle and impinging plates were made of plywood, but the sidewalls were made of Plexiglas in order to facilitate flow visualization of the jet oscillation. As shown in Figure 1, three loudspeakers were mounted on each side of the nozzle exit. The three speakers at each side were connected in series and constituted one set of speakers. Aluminium plates covered



Figure 1. Experimental set-up illustrating a two-dimensional jet impinging on a slot, and an acoustic resonator consisting of two deep cavities (all dimensions are in mm). 1: nozzle; 2: slot; 3: cavity; 4: plate forming the slot; 5: speaker cover plate; 6: side wall; 7: speaker.

the loud speakers and the side gaps were sealed, except those towards the nozzle lip, such that the excitation by the speakers is focused on the jet exit only. Further details of the test facility can be found in Ziada (1995).

A blower supplied airflow from outside the anechoic chamber and the flow rate was regulated by means of a variable speed motor. A silencer was positioned between the blower and the nozzle to attenuate the blower noise. The flow velocity at the nozzle exit, *V*, was calculated from the measured pressure drop across the nozzle. The pressure fluctuations were measured at three locations. As shown in Figure 1, microphones 1 and 2 were flush mounted at the closed ends of the cavities, and microphone 3 was located outside the cavity 10 cm from the centre of the slot and at 45° from the jet centreline. The signal of microphone 1 was phase-shifted, amplified and used to activate the speakers. Depending on the oscillation pattern, which was being controlled, the two sets of speakers were connected to operate either in-phase or out-of-phase with each other. It was therefore possible to impose *either* symmetric or antisymmetric excitation on the jet exit. However, *these two modes could not be imposed simultaneously*.

Visualization of the jet oscillation was achieved by injecting smoke into the nozzle before its exit. A strobe light was used to slow the jet oscillation which was then recorded on a video system. The photographs presented in this paper were produced by means of a video printer.



Figure 2. Characteristics of the system oscillation as the flow velocity is increased: (a) frequency of oscillation; (b) normalized pressure amplitude; (c) phase between pressure pulsations at locations 1 and 2.

3. JET-SLOT-CAVITY INTERACTION MECHANISMS

The frequency, f, and the dimensionless amplitude, P, of the dominant component of pressure oscillations are plotted in Figure 2 as functions of the flow velocity at the nozzle exit, V. The pressure amplitude is normalized by the jet dynamic head, i.e.,

$$P = P_{\rm rms} / (\frac{1}{2}\rho V^2).$$
 (1)

Here $P_{\rm rms}$ is the root-mean-square amplitude and ρ is the density. The results shown in Figure 2 were measured by means of microphone 1, and have been found to be almost identical to those obtained by microphone 2. The phase angle between microphones 1 and 2 is given in Figure 2(c). The resonance of the acoustic standing waves along the cavity depth, Figure 3(a), is seen to occur over two ranges of flow velocity, between V = 16 and



Figure 3. Schematic presentation of the jet oscillation patterns of the three investigated cases. f is the oscillation frequency; f_a is the acoustic resonance frequency; f_{js} is the frequency of the natural jet-slot oscillation and S is the Strouhal number.

19 m/s and at V > 24 m/s. Within these ranges, the oscillation frequency (≈ 292 Hz) agrees well with that calculated from the formula:

$$f = c/2 H, \tag{2}$$

where c is the sound speed and H the distance between the closed ends of the cavities (590 mm).

Outside the resonance ranges, where the flow oscillations resemble the natural oscillation mode of the jet-slot oscillator, the frequency increases smoothly with the flow velocity, and the pressure oscillations in the two cavities are in-phase. This suggests that the jet oscillates symmetrically when the acoustic resonance is not excited, Figure 3(b). This is illustrated to be indeed the case by the flow visualization photograph B1 in Figure 4. At the onset of resonance, the pressure amplitude increases sharply with flow velocity and the pressure oscillations in the cavities become out-of-phase with each other. Thus, the jet oscillation pattern switches to an antisymmetric mode when the resonance is excited, as illustrated schematically in Figure 3(a, c) and by means of flow visualization in Figure 4(A1 and C1). At the onset of the first resonance range ($V \approx 16$ m/s), the oscillation switches not only from a symmetrical to an antisymmetrical mode, but also to a higher mode of jet



Figure 4. Flow visualization of the jet oscillations with and without control for the non resonant Case B and the resonant Cases A and C.

oscillation, whereby the number of the formed vortices along the cavity mouth is increased, Figures 3(c) and 4(C1). The excitation of acoustic resonance by higher-order modes of the shear layer has been reported for several other cases, such as a pipeline with two baffles [e.g. Harris *et al.* (1988) and Hourigan *et al.* (1990)] and a pipe with closed side branches [e.g. Graf & Ziada (1992) and Ziada (1993)].

Another resonance phenomenon, which has been observed during the tests, is that associated with the Helmholtz resonator. In this case, the two cavities constitute the volume of the resonator and the nozzle together with the slot represent the resonator neck. An estimation of the Helmholtz resonance frequency (Kinsler *et al.* 1982) indicates that the resonance frequency is about 125 Hz. This clarifies the mechanism generating the hump in

the amplitude curve near V = 12 m/s in Figure 2, and the cause of the frequency jump which occurs near V = 7 m/s. As expected, the two microphones at the closed ends of the cavities remain in-phase with each other when the Helmholtz resonance is excited.

As indicated in Figure 2, three test conditions have been chosen to investigate the effect of feedback control on the system oscillation and in particular when the cavity resonance is excited. The first test-case is Case B, which is a typical example of the natural jet-slot oscillation in the absence of any resonance effects. The jet oscillates symmetrically at a Strouhal number of $S_B \approx 0.87$, where S is defined by

$$S = fL/V. \tag{3}$$

In this equation, L is the distance between the nozzle exit and the slot (85 mm).

Case A represents a very strong resonance case within the second lock-in range, where the resonance frequency is virtually equal to the natural oscillation frequency of the jet-slot oscillation. The Strouhal number of this case is $S_A \approx 0.86$, which is similar to that of the natural oscillations of Case B. However, the jet oscillation mode at resonance is different from that occurring at off-resonance conditions, as can be seen from the flow visualization photographs given in Figure 4. Case C is also a strong resonance case, but within the first lock-in range where the Strouhal number ($S_C \approx 1.43$) is substantially higher than that of the natural instability of the jet-slot system and the oscillation mode is different from the natural instability mode. The effect of feedback control in this case is of special interest because of the possibility that the system may revert to the natural oscillation mode of the jet-slot oscillator when the cavity resonance is suppressed.

Pressure spectra measured at locations 1 and 3 are given in Figure 5 for Cases B and C. Generally, the sound pressure level is substantially lower outside than inside the cavity, locations 3 and 1, respectively. Interestingly, when resonance is excited, Case C, the resonance peak at 292 Hz, which dominates the cavity pressure spectra, is hardly discernible outside the cavity. In contrast, in the nonresonant Case B, the frequency of the natural flow oscillations (220 Hz), and its sub- and higher superharmonics are clearly present in the spectra measured outside the cavity. This difference is due to the fact that, in Case C, the acoustic flux at the resonance frequency alternates between the two cavities and virtually no acoustic energy is radiated from the cavities into the surroundings, i.e. through the slot to microphone 3. For a similar geometry consisting of two coaxial side branches, Graf & Ziada (1992) have found that only 2% of the cavity acoustic energy is radiated from the cavities into the associated piping system. Finally, it is noteworthy that the spectra of the nonresonant Case B given in Figure 5 show a strong presence of the subharmonic component at 110 Hz. This subharmonic component seems to be enhanced because its frequency is close to that of the Helmholtz resonator.

4. FEEDBACK CONTROL OF THE NONRESONANT OSCILLATIONS

Since the jet oscillation in the nonresonant Case B is symmetric, the two sets of loudspeakers, at the opposite sides of the jet exit, were connected to operate in-phase with each other and therefore counteract the fluid-dynamic feedback, which has a symmetric phase distribution in this case. The speakers were activated by the signal of microphone 1. The phase of this signal was adjusted and then fed to a power amplifier which activated the speakers. Both the phase and gain of the signal activating the speakers were adjusted until the best performance was achieved. A similar approach was used in the resonant Cases A and C, but the two sets of speakers were operated out-of-phase with each other, to match the antisymmetric pattern of jet oscillations.



Figure 5. Pressure spectra measured inside and outside the cavity, by Mic. 1 and 3, at resonant oscillations, Case C, and nonresonant jet-slot oscillations, Case B: Without control —, Mic. 1; ……, Mic. 3.

Figure 6 shows typical pressure spectra recorded with and without control for Case B. The amplitude of the pressure pulsation at 220 Hz is reduced by about 19 dB, both inside and outside the cavities. Additionally, all the sub- and superharmonics are totally eliminated from the spectra. The elimination of these harmonics, although the controller is tuned to (i.e., acting upon) the fundamental component only, indicates that these harmonics are generated by nonlinear effects associated with large amplitude oscillations. These harmonics simply disappear when the amplitude of the fundamental component is reduced. The secondary peak near 125 Hz, which can be seen in the controlled spectrum of location 1, is the response of the Helmholtz resonator. Its amplitude increases as the dominant oscillation mode is attenuated. However, the amplitude of this peak is very small compared with that of the uncontrolled dominant peak (22 dB lower).

Flow visualization photographs of the nonresonant Case B, with and without control, are shown in Figure 5(Case B). When feedback control is applied, the very organized symmetric oscillation of the jet is eliminated and the mixing zone of the jet displays the features of incoherent, turbulent mixing layers. These results agree well with those reported by Ziada (1995) for the case of jet-slot oscillations in the absence of resonance effects.

5. FEEDBACK CONTROL OF THE JET-SLOT-CAVITY INTERACTION

As mentioned earlier, when resonance was excited, the loudspeakers were connected to operate out-of-phase with each other, to match the phase of the antisymmetric mode of jet



Figure 6. Pressure spectra measured inside and outside the cavity, Mic. 1 and 3, showing the effect of feedback control on the nonresonant oscillation, Case B:, without control; ..., with control.

oscillation. Figure 7 depicts the effect of feedback control on the pressure spectra measured at locations 1 and 3 for Case A, in which the resonance frequency approximates the natural frequency of the jet-slot oscillator. Activating the control system reduces the amplitude of acoustic resonance by an amount of 13 dB. Moreover, the secondary spectral peaks are either eliminated or substantially attenuated. Interestingly, the resonance amplitude outside the cavity, location 3, increases slightly when the controller is activated, although its counterpart inside the cavity is reduced by 13 dB. This is because, when active control is applied, the oscillation frequency increases slightly, and is therefore shifted away from the cavity resonance frequency. As mentioned earlier, this increases the acoustic radiation through the slot to microphone 3.

An active attenuation of the resonant Case C is clearly more difficult to achieve than the other two cases. The jet oscillates at a frequency which is substantially higher than the natural frequency of the jet-slot oscillator. Under these conditions, the suppression of resonance may destabilize the most unstable mode of the jet-slot oscillator, which would occur at a frequency near 190 Hz as can be seen from Figure 2(a). The spectra given in Figure 8 show that this is indeed the case. As the controller gain is increased, the resonance amplitude at 290 Hz decreases gradually, but at the same time, the natural mode of the



Figure 7. Pressure spectra for Case A measured inside and outside the cavity, Mic. 1 and 3, showing the effect of feedback control on acoustic resonance when $f = f_a \approx f_{is}$:, without control, with control.

jet-slot oscillator near 190 Hz is destabilized. As shown in Figure 8, the maximum reduction in the resonance amplitude that could be achieved without strongly exciting the jet-slot mode was about 11 dB. In fact, the pressure amplitude at location 3 becomes substantially higher (≈ 8 dB) with control, because acoustic radiation of the natural jet-slot oscillation at 190 Hz is much higher than that associated with resonant cavity oscillations at 290 Hz.

Photographs (A) and (C) in Figure 4 show the effect of control on the jet structure for the resonant cases. Here again, the organized nature of the jet oscillations is reduced drastically as a result of reducing the pressure amplitude by an amount of 11–13 dB. Figure 4 also shows that the jet structure of the controlled resonance cases, Photographs A2 and C2, is much less organized than that of the uncontrolled natural jet-slot oscillation (Photograph B1).

6. DISCUSSION

The results of the resonant cases demonstrate the necessity of employing multiple sensors (here microphones) to be able to assess properly the effect of active control on flow oscillations. For example, in the nonresonant Case B, both microphones 1 and 3 showed



Figure 8. Pressure spectra for Case C measured inside and outside the cavity, Mic. 1 and 3, showing the effect of feedback control on acoustic resonance when $f = f_a \neq f_{is}$:, without control, with control.

similar attenuation, but in the resonant Cases A and C, microphone 3 actually showed an increase in the pulsation amplitude, although the amplitude inside the cavity was reduced by an amount ranging between 11 and 13 dB. This observation does not detract from the effectiveness of the present control method, because, when the resonance is excited, Figure 5(a) shows that the pulsation amplitude outside the cavity is several orders of magnitude lower than that inside the cavity. However, one should be aware of the possibility that reestablishing the jet-slot oscillation and the resulting increase in radiation losses may excite other system components if their resonance frequencies are close to that of the jet-slot oscillator.

Although the amount of noise attenuation for the investigated resonant cases is comparable with that achieved for the nonresonant case, it may be considered unsatisfactory, and additional attenuation may still be desired. This limited success is due to the limited capabilities of the control system used in the present study. This simple controller cannot attenuate resonant and nonresonant oscillations simultaneously. Thus, when the cavity resonance is suppressed, the system reverts to the most unstable mode of the jet-slot oscillator. As mentioned earlier, the two sets of speakers can be operated in one mode only, either in-phase or out-of-phase. Moreover, the phase of the control signal can be adjusted properly only for a very narrow frequency range centred at the oscillation frequency. Since *the frequency and the mode shape* of the jet oscillation under resonance conditions can be very different from those of the most unstable mode of the natural jet-slot oscillation, it is necessary to connect some speakers to operate out-of-phase, to control the resonant oscillations, and connect the rest of the speakers to operate in-phase, to suppress the jet-slot mode which would essentially be initiated when the cavity resonance is suppressed. Each set of speakers can then be activated by a separate controller to facilitate the control of symmetric and antisymmetric modes, which occur at different frequencies. Moreover, the use of digital controllers would allow a proper control over the whole frequency range of interest. It is believed that such a technique would substantially increase the amount of attenuation in the resonant cases. This has been shown to be the case in an earlier investigation, where the most unstable modes of the jet-edge and jet-slot oscillations have been totally suppressed by means of a digital controller without destabilizing other modes (Ziada 1995).

7. CONCLUSIONS

The coupling between a jet-slot oscillator, which is excited by the fluid-dynamic mechanism, and a deep cavity oscillator, which is excited by the fluid-resonant mechanism, has been investigated experimentally. Strong acoustic resonance of the fundamental mode of the cavity has been observed over two ranges of flow velocity, corresponding to different ranges of Strouhal number. At the onset of the higher Strouhal number resonance, a substantial jump in the oscillation frequency occurs, from the frequency of the jet-slot oscillator to the cavity resonance frequency. Additionally, the jet oscillation switches from the symmetric mode of the jet-slot oscillator to an antisymmetric one. In the second resonance range, the resonance frequency approximates that of the jet-slot oscillator, but the jet oscillation switches from the symmetric to the antisymmetric mode.

A simple controller has been used to investigate the feasibility of suppressing the resonant oscillation by active means when an additional, nonresonant but robust flow oscillator is also present. Moderate attenuation levels, 11–13 dB, have been achieved in both resonance cases. Further attenuation could not be achieved, because the jet-slot oscillator is destabilized beyond this limit. It is believed that additional attenuation can be achieved by using two systems of loudspeakers activated by two independent controllers. This would allow simultaneous phase matching of the loudspeaker excitation to the symmetric and the antisymmetric oscillation modes of the jet.

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